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Investigating a Guided Inquiry-Based Mobile Augmented Reality Approach to Enhance Students' Mathematical Representation and Self-Efficacy

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ABSTRACT

Despite the growing adoption of Augmented Reality (AR) and Guided Inquiry Learning (GIL) in mathematics education, their integration within a mobile learning environment to enhance students' mathematical representation (MR) and self-efficacy (SE) remains underexplored. To address this gap, the present study investigated the effectiveness of Mobile Augmented Reality–Integrated Guided Inquiry Learning (MAR-GIL) in enhancing students' MR and SE compared with Guided Inquiry Learning (GIL) and Conventional Learning (CL). A quantitative quasi-experimental design with a non-equivalent control group was employed, involving 96 ninth-grade students from Yogyakarta, Indonesia, who were divided into three groups: MAR-GIL (n = 32), GIL (n = 32), and CL (n = 32). Data were collected using an MR test and an SE questionnaire, then analyzed using descriptive statistics, One-Way ANOVA, Tukey's HSD post hoc test, eta-squared effect size, and N-Gain analysis. The findings revealed that the MAR-GIL group achieved the greatest improvement in MR and SE and demonstrated greater effectiveness than the GIL and CL groups. These findings underscore the value of integrating AR with guided inquiry learning to support students' MR and SE, with implications for instructional design, curriculum development, teacher training, and technology-based educational policy.

KEYWORDS

Mobile augmented reality, Guided inquiry learning, Mathematical representation, Quasi-experimental design, Self-efficacy

INTRODUCTION

The development of digital technologies, particularly mobile learning and Augmented Reality (AR), has created new opportunities in mathematics education. Mobile learning enables flexible access to interactive resources that support both individual and collaborative learning (Nurdin et al., 2023; Rohendi & Wihardi, 2020). Meanwhile, AR allows students to visualize abstract mathematical objects in more concrete and interactive ways, particularly in geometry and spatial concepts (Rebollo et al., 2022). Previous studies have shown that AR can positively influence students' engagement, motivation, interest, and academic achievement (Chen, 2019; Elsayed & Al-Najrani, 2021; Mpiti et al., 2023; Sudirman et al., 2025). Nevertheless, many applications of AR in mathematics learning still focus primarily on technological appeal rather than on pedagogical effectiveness (Elsayed & Al-Najrani, 2021; Guntur & Setyaningrum, 2021). As a result, AR is often used merely as a visualization tool without sufficiently promoting deeper reasoning or representational understanding. Despite the potential of these technologies, many students continue to struggle with mathematics learning.

Mathematics remains a challenging subject for many middle school students, particularly when they must grasp abstract concepts and express mathematical ideas through multiple representations (Nugroho et al., 2025; Nugroho & Septianisha, 2025). Students often struggle to connect visual, symbolic, and verbal representations, resulting in fragmented conceptual understanding (Rahmawati et al., 2025; Saifiyah & Retnawati, 2019). In this context, mathematical representation (MR) is essential for helping students construct meaning, communicate reasoning, and solve mathematical problems effectively (Putra et al., 2024; Samsuddin & Retnawati, 2018). In addition, self-efficacy (SE) plays a crucial role in shaping students' persistence, effort, and willingness to engage in challenging mathematical tasks (Shone et al., 2024). Low SE may reduce students' persistence and negatively affect their representational performance (Nugroho et al., 2025). These challenges underscore the importance of learning approaches that support students' cognitive and affective development through meaningful technology integration.

One instructional approach that aligns well with technology-enhanced learning is Guided Inquiry Learning (GIL). GIL emphasizes active exploration while providing structured guidance and scaffolding throughout the learning process (Pedaste et al., 2015). Through guided inquiry activities, students are encouraged to develop multiple representations, test mathematical ideas, and explain their reasoning systematically. Although many studies have examined mobile learning, AR, and GIL separately in mathematics education, several research gaps remain. First, most studies have investigated these approaches separately, with limited attention to integrating mobile AR and GIL within a unified mathematics learning environment. Second, existing AR research has primarily focused on outcomes such as conceptual understanding, engagement, achievement, critical thinking, creativity, problem-solving, numeracy, spatial reasoning, self-efficacy, and reduced mathematics anxiety (Angraini et al., 2023; Chao & Chang, 2018; Chen, 2019; Elsayed & Al-Najrani, 2021; Makgakga, 2023; Nindiasari et al., 2024; Pramulia et al., 2025; Richardo et al., 2023), while its potential to enhance students' MR and SE has received relatively little attention. Similarly, studies on GIL have mainly emphasized reasoning, critical thinking, and problem-solving skills (Eshetu et al., 2022; Istikomah et al., 2022; Opticia et al., 2022; Simamora et al., 2018), but have rarely examined its integration with immersive technologies such as mobile AR.

Specifically, in middle school mathematics learning, the effectiveness of Mobile Augmented Reality–Integrated Guided Inquiry Learning (MAR-GIL) in improving both students' MR and SE remains underexplored. Therefore, it remains unclear whether MAR-GIL yields greater gains in students' MR and SE than GIL and conventional learning (CL). To address this issue, this study investigates the effectiveness of MAR-GIL in improving students' MR and SE. Accordingly, this study addresses the following research questions:

RQ1. How is MAR-GIL implemented in mathematics learning?

RQ2. Are there significant differences in students' MR among those who learn MAR-GIL, GIL, and CL?

RQ3. Are there significant differences in students' SE among those who learn MAR-GIL, GIL, and CL?

RQ4. How effective are MAR-GIL, GIL, and CL in improving students' MR and SE?

Research Hypothesis

Based on the research questions and previous empirical findings indicating that GIL improves students' cognitive skills, such as critical thinking and mathematical literacy, and learning achievement, as well as affective outcomes, such as self-efficacy (Prasanti et al., 2023; Pujiastuti & Haryadi, 2022; Vonitasari & Amir, 2025). Similarly, AR has been reported to enhance cognitive abilities, including mathematical achievement, representation, and spatial thinking, as well as affective aspects such as motivation and self-efficacy (Aldeeb et al., 2024; Guntur & Setyaningrum, 2021; Li et al., 2022). Based on these findings, this study hypothesizes that integrating AR and GIL can improve students' MR and SE. The hypotheses are as follows:

H_1: There is a significant difference in students' MR average scores across groups

H_2: There is a significant difference in students' SE average scores across groups

H_3: The MAR-GIL group has a higher average MR score than the GIL group

H_4: The MAR-GIL group has a higher average MR score than the CL group

H_5: The MAR-GIL group has a higher average score of SE than the GIL group

H_6: The MAR-GIL group has a higher average score of SE than the CL group

LITERATURE REVIEW

Cognitive Load Theory in Mathematics Learning

Cognitive Load Theory (CLT), proposed by Sweller (1994), holds that learning is most effective when instructional design optimizes the limited capacity of learners' working memory by managing intrinsic (the inherent complexity of the learning material), extraneous (the unnecessary cognitive effort caused by ineffective instructional design), and germane (the cognitive effort devoted to constructing and refining knowledge structures) cognitive load. In mathematics learning, students often process multiple representations simultaneously, such as symbols, diagrams, graphs, and verbal explanations, which can increase cognitive demands and hinder conceptual understanding (Bokosmaty et al., 2015). For example, when solving geometry problems, students frequently need to mentally transform two-dimensional representations into three-dimensional objects while interpreting mathematical symbols and verbal information. Such processes may overload working memory and hinder conceptual understanding. From the perspective of CLT, AR can reduce unnecessary extraneous cognitive load by presenting abstract mathematical concepts as interactive three-dimensional visualizations, allowing students to manipulate mathematical objects directly rather than relying solely on mental visualization. Consequently, students can allocate more cognitive resources to constructing mathematical understanding and connecting visual, symbolic, and verbal representations.

Theory of Mathematical Representation for Mathematics Education

The theory of mathematical representation views representation as a fundamental means by which learners construct, communicate, and interpret mathematical ideas. According to Villegas et al. (2009), mathematical understanding develops through students' ability to use and translate among multiple forms of representation. In mathematics learning, MR is commonly classified into three major aspects: visual, symbolic, and verbal representation, following the framework proposed by Villegas et al. (2009). For example, when solving a three-dimensional geometry problem, students may draw the geometric object (visual representation), formulate the corresponding mathematical equations to calculate its surface area or volume (symbolic representation), and explain the solution process and reasoning in written or spoken form (verbal representation). These aspects are presented in Table 1.

Table 1.

MR Indicator

MR Aspects	Description
Visual Representation	The ability to represent mathematical ideas using diagrams, graphs, or tables
Symbolic Representation	The ability to express mathematical relationships through symbols or equations
Verbal Representation	The ability to explain mathematical reasoning clearly and logically

Social Cognitive Theory of Self-Efficacy in Mathematics Learning

According to Bandura’s (1997) social cognitive theory, SE is an individual’s belief in their ability to accomplish specific tasks. In mathematics education, SE influences students’ persistence, effort, resilience, and willingness to engage with challenging problems (Nugroho et al., 2025; Sibanda, 2023). Students with strong SE tend to apply a variety of solution strategies, correct mistakes, and remain engaged in challenging activities, thereby supporting the development of

MR skills. In contrast, students with low SE are more likely to avoid difficult tasks, hesitate to communicate their reasoning, and give up easily, which may limit their representational abilities. For example, students with high SE are more willing to solve unfamiliar geometry problems, explain their reasoning during class discussions, and persist until they find a correct solution, whereas students with low SE tend to avoid challenging tasks and rely heavily on teacher assistance. Bandura (1997) identifies SE through three main dimensions, as outlined in Table 2.

Table 2.

SE Indicator

SE Aspects	Description
Magnitude	Belief in the ability to complete tasks of varying difficulty
Strength	The level of confidence in one's ability to perform a task.
Generality	The extent to which self-efficacy applies across different situations or tasks.

Mobile Augmented Reality in Mathematics Education

AR integrates digital content with the real-world environment in real time (Dargan et al., 2023). Unlike Virtual Reality, which creates a fully simulated environment, AR overlays interactive virtual elements onto physical surroundings via devices such as smartphones and tablets (Crogman et al., 2025). In mathematics education, AR is widely used to support the visualization of abstract concepts that are difficult to grasp through conventional instruction (Özdemir & Özçakır, 2026). Using AR in learning is associated with benefits such as increased engagement, motivation, conceptual understanding, and mathematical representation skills (Cevikbas et al., 2023). Educational AR applications are generally classified as marker-based AR, which uses images or QR codes to trigger digital content, and markerless AR, which relies on sensors and object recognition without physical markers. In mathematics learning, AR supports both cognitive and affective outcomes (Aldeeb et al., 2024; Elsayed & Al-Najrani, 2021; Guntur & Setyaningrum, 2021; Pramulia et al., 2025).

Guided Inquiry Learning as an Instructional Framework

Table 3.

Overview of GIL Phases

GIL Phases	Description
Orientation	Students are introduced to a problem to activate prior knowledge and curiosity
Conceptualization	Students develop questions and initial ideas about the problem
Investigation	Students explore, collect data, and interpret findings
Conclusion	Students draw conclusions based on evidence
Discussion	Students share results, receive feedback, and reflect on learning

Guided Inquiry Learning (GIL) integrates two core concepts: inquiry and guidance. Inquiry emphasizes investigation, questioning, and active knowledge construction, whereas guidance denotes the structured support teachers provide during learning (Mabhoza & Olawale, 2024; Pedaste et al., 2015). According to Chase et al. (2013), inquiry-based learning encourages students to construct understanding through exploration rather than passively receiving information. Consequently, learning becomes a process of investigation and meaning-making. Building on established inquiry-learning frameworks, GIL can be organized into five main phases, as shown in Table 3.

The Potential Framework of Mobile Augmented Reality–Integrated Guided Inquiry Learning to Enhance Students' Mathematical Representation and Self-Efficacy

The integration of MAR-GIL has the potential to create a more meaningful and interactive mathematics learning environment that supports students' MR and SE. AR enables students to interact with three-dimensional objects, making abstract concepts more concrete and supporting visual, symbolic, and verbal representations through direct manipulation of mathematical models (Li et al., 2022; Shaghaghian et al., 2022). Meanwhile, guided inquiry encourages students to actively explore problems, construct ideas, test relationships, and communicate reasoning through discussion and reflection (Istikomah et al., 2022). These activities strengthen conceptual understanding and representation skills while also fostering confidence in problem-solving. The immersive visualization and feedback from AR reduce students' uncertainty in learning abstract concepts (Pujiastuti & Haryadi, 2022), while repeated inquiry experiences enhance self-efficacy through mastery and collaboration (Erbilgin et al., 2024; Simamora et al., 2018). Overall, MAR-GIL is expected to support both cognitive and affective learning outcomes more effectively than conventional instruction.

METHODOLOGY

Research Design

This study used a quantitative quasi-experimental design with a non-equivalent control group, which is suitable when random assignment cannot be implemented (Creswell, 2014). This design enabled the researchers to compare three instructional models: (1) MAR-GIL as Experimental Group 1 ($T_{1)}$, (2) GIL as Experimental Group 2 ($T_{2)}$, and (3) CL as the control group (X). The research design is presented in Table 4. All groups received a pre-test to assess initial equivalence and a post-test to measure the effects of the treatments on students' MR and SE.

Table 4.

Research Design of the Study

Sample Groups	Pre-tests	Treatment	Post-test
MAR-GIL	✓	T_1	✓
GIL	✓	T_2	✓
CL	✓	X	✓

Research Participants

Participants were Grade 9 students (13–14 years old) from a private junior high school in Yogyakarta, Indonesia. Of 11 available classes, students were not randomly assigned to experimental conditions. Instead, cluster random sampling was used to select three intact classes. Each selected class had 32 students, for a total of 96 participants. The classes were assigned to the MAR-GIL, GIL, and CL groups while maintaining intact classroom settings. Participation was voluntary, and informed consent was obtained before data collection. The school was chosen for its adequate technological infrastructure and students' familiarity with mobile devices, which supported the use of mobile AR. Additionally, students had not previously studied three-dimensional geometry, ensuring relatively equivalent prior knowledge across the groups.

Research Instruments

This study used two main instruments to assess students' MR and SE. The MR instrument was an essay-based mathematics test with four open-ended questions on three-dimensional geometry, administered as both the pre-test and post-test. The same items were used in both tests to ensure comparability of students' performance. Each item was designed to measure three aspects of MR: visual, symbolic, and verbal representations, as presented in Table 5. The items presented contextual problems in three-dimensional geometry and required students to demonstrate integrated mathematical representations in each response. Students were given 90 minutes to complete the test, with a maximum score of 100.

Table 5.

Blueprint of the MR Instrument

Indicator	Item Numbers
Students are presented with a real-life problem involving a cube with circular holes and are asked to determine the surface area of the cube	1
Students are presented with a real-life problem about filling a rectangular aquarium with water and are asked to determine the volume of water	2
Students are presented with a real-life problem about wrapping a prism-shaped piggy bank and are asked to determine its surface area	3
Students are presented with a real-life problem involving a pyramid-shaped lantern and are asked to determine its volume	4

The SE instrument consisted of a 20-item questionnaire administered during both the pre-test and post-test. It included both positive and negative statements to minimize response bias and strengthen construct validity. The items were developed based on three dimensions of SE— magnitude, strength, generality —as presented in Table 6. Positive items measured students' confidence, persistence, and belief in their ability to solve mathematical problems, whereas negative items reflected doubt and perceived difficulty. Responses were recorded on a Likert scale from 1 (strongly disagree) to 5 (strongly agree), and students were allotted 45 minutes to complete the questionnaire.

Table 6.

Blueprint of the SE Questionnaire

Indicator	Item Numbers
Confidence in performing or completing mathematical tasks and problems with varying levels of difficulty	3, 5, 6, 8, 9, 12, 15
Confidence in applying mathematical abilities across different situations and conditions	1, 7, 14, 17, 19, 20
Strength of belief and persistence in making efforts to solve mathematical tasks	2, 4, 10, 13, 16, 18

Before data collection, two mathematics education experts from a public university in Yogyakarta, Indonesia, reviewed both instruments using a five-point Likert scale to assess item relevance, clarity, and suitability. Content validity was evaluated using Aiken's V, yielding high validity scores of 0.87 for the MR instrument and 0.92 for the SE instrument. After minor revisions based on expert suggestions, practicality testing was conducted with 86 junior high school students. Reliability analysis using Cronbach's alpha indicated acceptable internal consistency, with coefficients of 0.75 for MR and 0.77 for SE.

Research Procedures

The study was conducted over 10 sessions. The first meeting was used to administer the pre-test to measure students' initial MR and SE, and the last meeting was used to administer the post-test. The eight middle were used to implement the instructional treatments, each lasting 90 minutes, with the same mathematics teacher teaching all groups. The learning material focused on three-dimensional geometry, including cubes, rectangular prisms, and pyramids. The overall data collection process spanned approximately 7 months (October 2025–April 2026), covering instrument development, validation, implementation, and analysis.

In the MAR-GIL group, learning followed an integrated AR and GIL syntax. Students were introduced to contextual problems supported by AR visualizations, then explored 3D objects through scanning, manipulation, and guided inquiry activities, including observing, analyzing, and constructing MR. Students also presented findings and engaged in discussions to strengthen understanding and SE (Figure 1).

In the GIL group, the instructional process followed the same inquiry phases without AR technology. The orientation phase used contextual problems in worksheets to stimulate curiosity. In the conceptualization phase, students formulated questions and hypotheses in response to guided prompts. The investigation phase involved paper-based activities, discussions, and worksheet-guided exploration. In the conclusion phase, students derived mathematical concepts from their findings, and the discussion phase involved presenting results, problem-solving, and clarifying misconceptions through peer and teacher feedback

(Figure 2). In contrast, the CL group received conventional teacher-centered instruction, in which the teacher explainded concepts, provides examples, and assigned practice exercises, with limited opportunities for inquiry and interaction.

Figure 1.

The Syntax of MAR-GIL

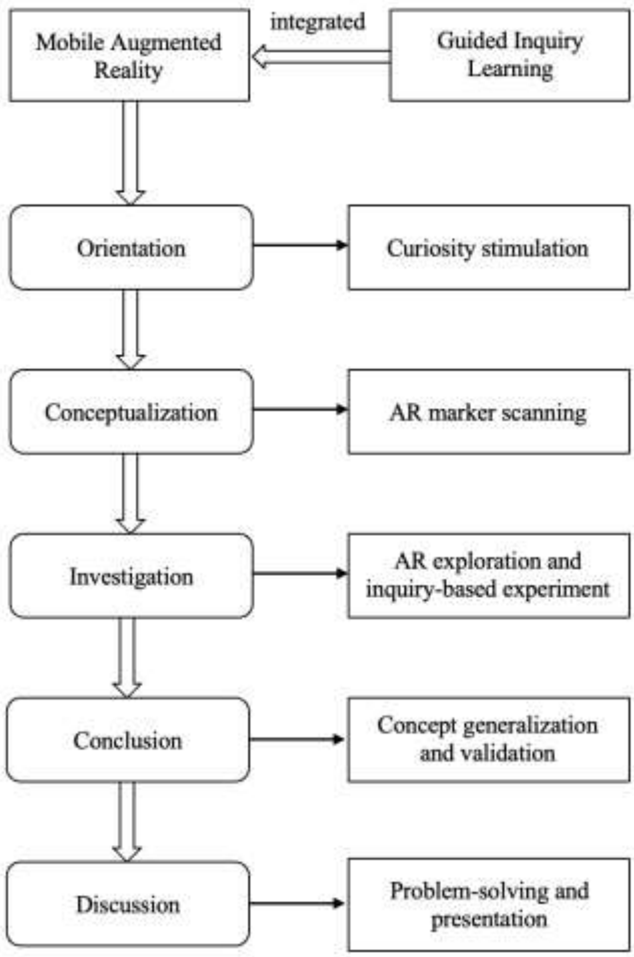
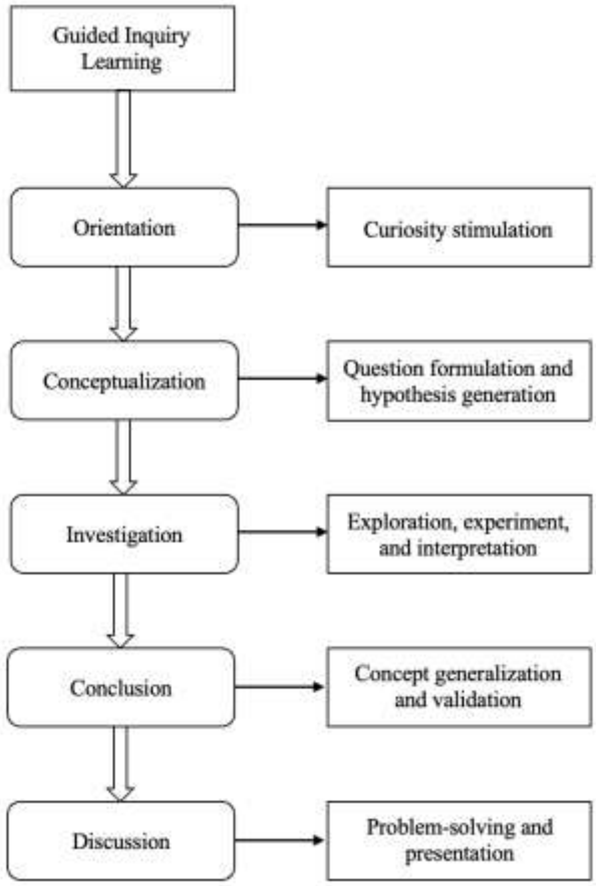


Figure 2.

The Syntax of GIL



Data Analysis Techniques

Data were analyzed in RStudio. Descriptive statistics summarized students' MR and SE scores across groups. Before inferential testing, ANOVA assumptions—normality and homogeneity of variance—were examined. A One-Way ANOVA was then applied to compare MR and SE among the MAR-GIL, GIL, and CL groups, followed by Tukey's HSD post hoc test when significant differences emerged. Effect size was calculated using eta-squared (η^2), interpreted according to Abbott (2014), with values of 0.01 categorized as small, 0.06 as medium, and 0.15 as large. To assess the effectiveness of each treatment, normalized gain (N-Gain) scores were computed to measure improvement from pre-test to post-test. Based on Hake (1999), N-Gain scores below 40% indicate ineffective improvement, 40–55% indicate less effective improvement, 56–75% indicate moderately effective improvement, and above 76% indicate highly effective improvement.

FINDINGS

Implementation of Mobile Augmented Reality-Integrated Guided Inquiry Learning

The implementation of MAR-GIL uses the AR-assisted mobile application "EdgeWander" for three-dimensional geometry topics, including cubes, rectangular prisms, prisms, and pyramids (see Figure 3). The learning process follows the phases of GIL: orientation, conceptualization, investigation, conclusion, and discussion, with students working collaboratively in small groups.

Figure 3.

Main Page Display of EdgeWander

During the orientation phase, students observe contextual examples of geometric solids and explore AR-based 3D objects by scanning markers using the EdgeWander application. Students initially produce simple sketches of the objects. After interacting with the AR models from multiple perspectives, they begin to represent geometric elements such as faces, edges, and vertices more accurately (see Figure 4).

Figure 4.

Students Observe and Explore EdgeWander Application

During the conceptualization and investigation phases, students manipulate AR objects to examine geometric relationships, explore nets, and analyze surface area and volume (see Figure 5). Through direct interaction and inquiry-based activities, students develop a deeper understanding of abstract three-dimensional concepts that are often difficult to visualize with static materials. Collaborative discussions are also observed, with students exchanging ideas, justifying mathematical relationships, and using mathematical terminology during problem-solving activities.

Figure 5.

AR-Based Cube Visualization in the Conceptualization Phase

During the conclusion and discussion phases, students wrap up their investigations and present their findings in class discussions (see Figure 6). Most students demonstrate improved understanding by explaining formulas and geometric relationships based on their exploratory experiences rather than by memorization. In addition, students show increased confidence in communicating mathematical ideas, asking questions, and defending their arguments during collaborative discussions.

Figure 6.

Group Presentation and Collaborative Discussion Activities

Pre-test Data Analysis Results

The pre-test analysis assesses students' initial abilities in MR and SE before the instructional treatment. The Shapiro–Wilk test results indicate that all p-values exceed 0.05, confirming that the pre-test data are normally distributed (see Table 7). In addition, Levene's test indicates homogeneous variances for MR (p = 0.615) and SE (p = 0.905) (see Table 8). Therefore, the assumptions for One-Way ANOVA are met.

Table 7.

Pre-test Data Normality Test Results

Sample Groups	Variable Statistic	p-value
MAR-GIL	MR	0.687
	SE	0.469
GIL	MR	0.113
	SE	0.111
CL	MR	0.089
	SE	0.365

Table 8.

Results of the Homogeneity of Variance Test for Pre-test Data

Levene Statistic	df_1	df_2	p-value
MR	2	92	0.615
SE	2	92	0.905

The One-Way ANOVA results indicate no significant differences among the MAR-GIL, GIL, and CL groups for MR (p = 0.897) and SE (p = 0.408) (see Table 9). This finding is consistent with the descriptive statistics in Table 10, which show similar mean scores across groups. Thus, all groups are considered to have equivalent initial abilities before the instructional treatment.

Table 9.

One-Way ANOVA Test Results for Pre-tests Data

Variable	Condition	df	Sum of Squares	Mean Square	F	p-value
MR	Model	2	29	14.38	0.109	0.897
	Residuals	92	12115	131.68		
SE	Model	2	196	97.95	0.906	0.408
	Residuals	92	9949	108.14		

Table 10.

Descriptive Statistical Test Results for Pre-tests Data

Variable	Sample Groups	Mean	Minimum	Maximum	SD
MR	MAR-GIL	47.500	20	75	12.378
	GIL	48.125	30	70	10.219
	CL	46.774	20	65	11.729
SE	MAR-GIL	45.750	25	65	9.847
	GIL	42.375	20	60	10.376
	CL	44.871	25	70	10.963

Post-test Data Analysis Results

The post-test analysis examines differences in students' MR and SE after the instructional treatment. The results of the normality and homogeneity tests indicate that all assumptions are met, as all p-values exceed 0.05 (see Tables 11 and 12).

Table 11.

Post-test Data Normality Test Results

Sample Groups	Variable Statistic	p-value
MAR-GIL	MR	0.082
	SE	0.269
GIL	MR	0.063
	SE	0.070
CL	MR	0.307
	SE	0.101

Table 12.

Results of the Homogeneity of Variance Test for Post-test Data

Levene Statistic	df1	df2	p-value
MR	2	92	0.098
SE	2	92	0.418

Once both assumptions are satisfied, the analysis continues with One-Way ANOVA. According to Table 13, significant differences are observed among the groups for both MR ($F = 16.97$, $p < 0.001$) and SE ($F = 44.11$, $p < 0.001$). These findings suggest that H_1 and H_2 are supported. The η squared values for MR and SE are 0.27 and 0.49, respectively, meaning that 27% of the variance in students' MR and 49% of the variance in students' SE are explained by differences in instructional treatments (MAR-GIL, GIL, and CL). These effect sizes are considered large.

Table 13.

One-Way ANOVA Test Results for Post-test Data

Variable	Condition	df	Sum of Squares	Mean Square	F	p-value
MR	Model	2	3503	1751.6	16.97	< 0.001
	Residuals	92	9495	103.2		
SE	Model	2	3663	1831.7	44.11	< 0.001
	Residuals	92	3821	41.5		

Since significant differences are observed, a post hoc Tukey's HSD test is performed to pinpoint the disparities among the groups. According to Table 14, all pairwise comparisons for MR reveal significant differences ($p < 0.05$). The MAR-GIL group records notably higher MR scores than the GIL and CL groups, with the GIL group also outperforming the CL group.

Descriptive statistics in Table 16 show MAR-GIL with the highest mean MR score (83.906), followed by GIL (75.625) and CL (69.032). These results suggest that MAR-GIL yields the greatest MR improvement. Consequently, H_3 and H_4 are confirmed.

Table 14.

Tukey HSD Test Results for MR Data

Sample Groups for MR Data	Comparison	Mean Diff.	Lower	Upper	p-value	Description
MAR-GIL	GIL	8.281	2.231	14.332	0.004	Significant
	CL	14.874	8.775	20.973	0.000	Significant
GIL	MAR-GIL	-8.281	-14.332	-2.231	0.004	Significant
	CL	6.593	0.494	12.692	0.031	Significant
CL	MAR-GIL	-14.874	-20.973	-8.775	0.000	Significant
	GIL	-6.593	-12.692	-0.494	0.031	Significant

Meanwhile, the Tukey HSD results for the SE variable show significant differences across all group comparisons. The MAR-GIL group achieves significantly higher SE scores than the GIL and CL groups, and the GIL group also outperforms the CL group. Descriptive statistics in Table 16 indicate that MAR-GIL has the highest mean SE score (81.844), followed by GIL (71.968) and CL (66.871). These findings indicate that MAR-GIL is the most effective instructional model for improving students' SE. Therefore, H_5 and H_6 are accepted.

Table 15.

Tukey HSD Test Results for SE Data

Sample Groups for SE Data	Comparison	Mean Diff.	Lower	Upper	p-value	Description
MAR-GIL	GIL	9.875	6.037	13.713	0.000	Significant
	CL	14.973	11.104	18.842	0.000	Significant
GIL	MAR-GIL	-9.875	-13.713	-6.037	0.000	Significant
	CL	5.098	1.229	8.967	0.006	Significant
CL	MAR-GIL	-14.973	-18.842	-11.104	0.000	Significant
	GIL	-5.098	-8.967	-1.229	0.006	Significant

Table 16.

Descriptive Statistical Test Results for Post-test Data

Variable	Sample Groups	Mean	Minimum	Maximum	SD
MR	MAR-GIL	83.906	60	100	9.481
	GIL	75.625	50	95	12.240
	CL	69.032	50	85	8.308
SE	MAR-GIL	81.844	65	88	5.697
	GIL	71.968	56	83	7.293
	CL	66.871	55	76	6.233

Effectiveness Analysis of MAR-GIL, GIL, and CL

Finally, the effectiveness of each instructional treatment is evaluated using N-Gain analysis. The MAR-GIL group achieves the highest improvement, with N-Gain scores of 69.17% for MR and 64.85% for SE, both classified as moderately effective. The GIL group shows lower gains, with 51.68% for MR and 50.69% for SE, classified as less effective. Meanwhile, the CL group shows the lowest improvement, with 40.85% for MR and 37.06% for SE, falling into the less-effective to ineffective categories. Overall, these findings demonstrate that MAR-GIL is more effective than GIL and CL in improving students' MR and SE.

Table 17.

N-Gain Effectiveness Test Results

Treatment	Variable	Average of N-Gain Score	Category
MAR-GIL	MR	69.17%	Moderately effective
	SE	64.85%	Moderately effective
GIL	MR	51.68%	Less effective
	SE	50.69%	Less effective
CL	MR	40.85%	Less effective
	SE	37.06%	Ineffective

Qualitative Evidence of Students' Mathematical Representation

To complement the quantitative findings, representative student responses to one of the four post-test essay questions were analyzed. The selected responses illustrate how students in the MAR-GIL, GIL, and CL groups demonstrated visual, symbolic, and verbal skills in solving a contextual three-dimensional geometry problem. One of the post-test essay questions is presented below:

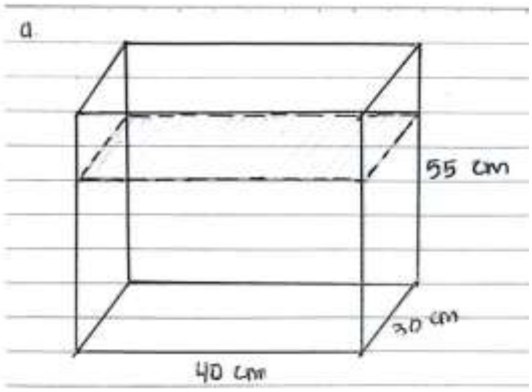
Arya wants to fill a rectangular aquarium with water until it is ¾ full. The dimensions of the aquarium are 40 cm × 30 cm × 55 cm. Determine the volume of water that Arya will pour into the aquarium by following the instructions below: (a) Draw a diagram of the aquarium filled to ¾ capacity; (b) Write a complete solution to find the answer; and (c) Provide your opinion on the obtained solution

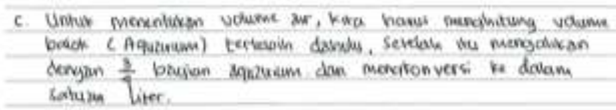
As shown in Figure 7, students in the MAR-GIL group successfully solve the problem by integrating visual and symbolic representations. Their responses demonstrate correct construction of the geometric model, accurate calculation of the volume of a rectangular prism, and appropriate use of the "3/4" proportion. They also explain the solution strategy by calculating the volume, multiplying it by "3/4", and converting the result to liters.

In the GIL group (Figure 8), students show partial understanding of the relationship between context and mathematical representation. Some students represent the water level as "1/2" rather than "3/4", indicating difficulty translating proportional information into spatial form. They correctly calculate the total volume of the rectangular prism ($p \times l \times t$ = 6600 cm³), but misunderstand the concept when determining water volume by “subtracting 3/4” instead of multiplying by “3/4”.

Figure 7.

Sample Student Answer in MAR-GIL Group

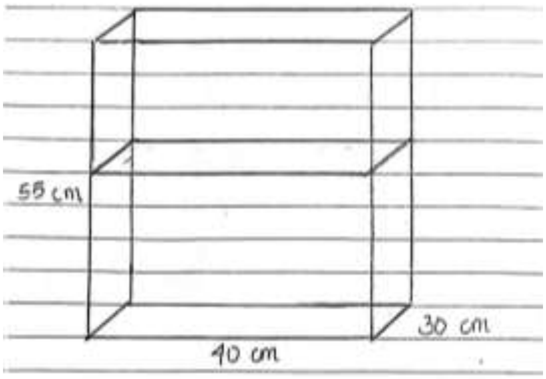


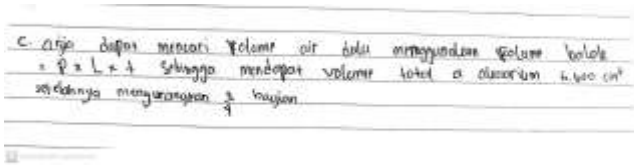


c. To determine the volume of water, we first need to calculate the volume of the rectangular prism (aquarium). Then, we multiply it by ¾ to represent the filled portion of the aquarium, and finally convert the result into liters.

Figure 8.

Sample Student Answer in GIL Group

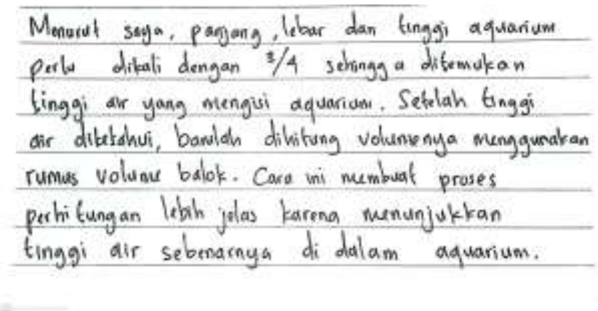




Arya first determines the volume of the water by calculating the volume of the rectangular prism using $p \times l \times t$, resulting in a total aquarium volume of 6600 cm³. Afterwards, he subtracts 3/4 of the volume.

Figure 9.

Sample Student Answer in CL Group



In my opinion, the length, width, and height of the aquarium should first be multiplied by ¾ to determine the height of the water filling the aquarium. After the water height is known, the volume can then be calculated using the rectangular prism volume formula. This method makes the calculation process clearer because it shows the actual height of the water inside the aquarium

Figure 9 presents a representative response from the CL group. The student interpreted the three-quarter condition by multiplying the aquarium's length, width, and height by "3/4" before calculating the volume. Although the student attempted to apply a mathematical procedure, the solution reflected an incorrect interpretation of the proportional relationship and did not yield the correct volume of water.

DISCUSSION

This study examines the effectiveness of MAR-GIL in improving students' MR and SE compared with GIL and CL. Overall, MAR-GIL yields the highest achievement and learning gains in both MR and SE and is classified as moderately effective. GIL performs moderately but less effectively, while CL yields the lowest results and is classified as less effective to ineffective. These findings indicate that MAR-GIL is the most effective approach among the three instructional models.

Comparison of the Effectiveness of MAR-GIL, GIL, and CL in Enhancing Students' MR Skills

The findings indicate that MAR-GIL is the most effective instructional approach for improving students' MR skills. Students in the MAR-GIL group outperformed those in the GIL and CL groups, suggesting that integrating AR into guided inquiry more effectively supports students' understanding and expression of mathematical ideas through visual, symbolic, and verbal representations. In visual representation, MAR-GIL helps students develop a stronger spatial understanding of geometric solids. The interactive AR environment enables students to observe and manipulate three-dimensional objects directly, making abstract concepts more concrete and accessible. This finding aligns with previous studies showing that AR enhances conceptual understanding through interactive mathematical representations (Elsayed & Al-Najrani, 2021; Guntur & Setyaningrum, 2021).

Regarding symbolic representation, the inquiry process within MAR-GIL encouraged students to construct mathematical formulas grounded in conceptual understanding rather than procedural memorization. Through guided exploration, students connected geometric relationships to symbolic expressions for surface area and volume. This process helped students translate visual information into mathematical symbols and formulas, which is essential for developing flexible mathematical understanding (Arefaine et al., 2022; Li et al., 2022; Nugroho & Septianisha, 2025; Shaghaghian et al., 2022).

MAR-GIL also strengthens verbal representation through collaborative discussions. Students are encouraged to explain observations, justify mathematical relationships, and communicate ideas using mathematical language. This interaction supports conceptual understanding and reasoning, consistent with Vygotsky's (1978) view of social learning. In addition, AR visualization reduces cognitive load by allowing geometric relationships to be observed directly rather than mentally imagined, enabling students to focus more on reasoning and representation processes (Sweller, 1994). As a result, students can better focus on reasoning and on forming mathematical representations.

Although the GIL group outperforms the CL group, its effectiveness remains lower than MAR-GIL's. As shown in Figure 8, students can perform procedural calculations but still struggle to connect proportional concepts to spatial representations. Without immersive visualization, students must mentally construct three-dimensional relationships, increasing cognitive load and limiting the development of visual and symbolic representations. In contrast, AR in MAR-GIL enables students to directly observe and manipulate geometric objects, making spatial and proportional relationships more concrete. This finding aligns with previous studies showing that inquiry-based learning improves reasoning and problem-solving but requires visual scaffolding to strengthen representational development (Ariani et al., 2022; Opticia et al., 2022).

Meanwhile, the CL group shows the least improvement in MR skills, as shown in Figure 9. Conventional instruction emphasizes procedural execution, leading students to rely more on procedures than on conceptual understanding. This suggests difficulty integrating visual, symbolic, and verbal representations. Limited exploration and representational flexibility in CL may lead to fragmented understanding and weaker representational competence (Nugroho et al., 2026).

Comparison of the Effectiveness of MAR-GIL, GIL, and CL in Enhancing Students' SE Skills

The Tukey HSD analysis shows that all instructional models differ significantly in their effects on students' self-efficacy (SE). MAR-GIL is significantly more effective than both GIL and CL, and GIL also significantly outperforms CL. These findings confirm that integrating augmented reality into MAR-GIL provides an additional advantage for improving SE compared with inquiry-based learning without AR.

The higher SE in the MAR-GIL group can be explained by AR's interactive features and immediate visual feedback, which reduce uncertainty when learning abstract mathematical concepts (Chan et al., 2025; Shodikin et al., 2025). In terms of magnitude, AR helps students feel more capable of solving mathematical tasks by making abstract concepts more concrete and easier to understand. In terms of strength, repeated success in inquiry and exploration activities strengthens students' confidence in their mathematical abilities, consistent with Bandura's (1997) theory. In terms of generality, students become more confident in applying their understanding to a range of mathematical tasks, such as interpreting representations, constructing formulas, and communicating mathematical ideas. In addition, Mayer & Moreno's (2016) Cognitive Theory of Multimedia Learning supports the idea that combining visual and verbal information enhances understanding.

Although GIL also shows a significantly higher SE than CL, its improvement remains lower than MAR-GIL's. Guided inquiry promotes participation, collaboration, and problem-solving, which support students' confidence through mastery experiences and social persuasion (Simamora et al., 2018; Urwatul Wutsqa et al., 2026; Vonitasari & Amir, 2025). However, without AR support, students must visualize abstract geometric relationships mentally, increasing cognitive demand and potentially reducing confidence, particularly among students with weaker spatial abilities. In contrast, AR in MAR-GIL reduces ambiguity by enabling direct observation and manipulation of virtual objects, thereby strengthening students' understanding and confidence in solving mathematical problems (Newman & DeCaro, 2019; Shengqiang et al., 2025).

The lower SE in the CL group is explained by its teacher-centered approach, which limits opportunities for active participation and independent problem-solving. Students have fewer opportunities to achieve mastery, which weakens their confidence in handling mathematical tasks (Damrongpanit, 2019; Jameson & Fusco, 2014; Nugroho et al., 2025; Nugroho et al., 2024). Moreover, limited interaction and feedback reduce opportunities for social persuasion and reflective learning, both of which are important for strengthening SE. As a result, students tend to rely heavily on teacher guidance and show lower confidence when facing unfamiliar or challenging problems.

Implications, Novelty, and Recommendations for Future Research

Theoretically, the findings support MAR-GIL as an effective approach to enhancing MR and SE by integrating visualization and structured inquiry, thereby highlighting the link between cognitive and affective learning processes. Practically, MAR-GIL can serve as an alternative instructional strategy to help students understand abstract concepts through interactive, inquiry-based learning. These findings also underscore the importance of developing pedagogically sound, technology-enhanced learning environments, along with supporting digital infrastructure and teacher professional development. For researchers, this study provides a basis for further investigation of immersive technologies and inquiry-based approaches in mathematics education. Furthermore, the findings contribute to Sustainable Development Goal 4 (SDG 4) by promoting quality education through technology-enhanced learning.

The novelty of this study lies in proposing and empirically validating MAR-GIL as a unified instructional model in which mobile AR is an integral component throughout the guided inquiry process. This study also provides empirical evidence that integrating mobile AR with guided inquiry can simultaneously enhance students' MR and SE, extending previous research that has predominantly examined these outcomes separately.

While this study provides valuable insights into the effectiveness of MAR-GIL in improving students' MR and SE, several areas remain to be explored. First, the implementation of MAR-GIL is limited to Android-based smartphones, which may restrict access for students without compatible devices. This limitation highlights potential challenges related to equity and the broader scalability of technology-integrated learning environments. Second, this study primarily employed a quantitative approach to evaluate students' learning outcomes. Therefore, future research is recommended to incorporate qualitative approaches to explore in greater depth the responses of both students and teachers, thereby providing a more comprehensive understanding of the learning process and the effectiveness of MAR-GIL.

CONCLUSION

This study examines the effectiveness of MAR-GIL in improving students' MR and SE compared with GIL and CL. The results show that MAR-GIL moderately improves both MR and SE, whereas GIL yields less improvement. In contrast, CL yields less improvement in MR and no improvement in SE. These findings indicate that integrating mobile augmented reality with guided inquiry learning provides stronger support for both cognitive and affective outcomes in mathematics learning. The contribution of this study is the empirical validation of MAR-GIL as an integrated technology-enhanced learning model that connects immersive visualization with inquiry-based learning to support both cognitive and affective outcomes. However, the findings are limited by the use of Android-based devices and the quantitative research approach conducted in a single school context. Future research should investigate MAR-GIL across diverse educational settings and employ mixed-methods approaches to further understand its implementation process and learning impacts.

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